On Turbulent Flow with Streamwise Rotation (Statistical Nature of Turbulence and its Dynamical Description based on coherent structures)

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On Turbulent Flow with Streamwise Rotation

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Abstract

The flow in rotating passages attracts great interests in the field of turbomachines. The flow in such devices is subjected to Coriolis and centrifugal forces resulted from general rotating vector. Since much efforts have done in the classical spanwise rotating channels, this study has investigated the turbulent channel flow with system rotation in the streamwise direction, in which the main rotating vector coincides with the flow direction, aiming at generalizing the system of rotation. Direct numerical simulation has been performed using spectral methods for a wide range of rotation number Ro_τ up to 15 and a Reynolds number of 150. Low rotation number shows slight effect in turbulence characteristics of that of plane channel without system rotation. No laminarization features have been captured even in high numbers that differs from the classical spanwise rotation case. Both walls attain the same level of turbulence intensities. The simulation shows nearly symmetric profiles for the mean streamwise velocity around the centre of the channel. More augmentation in the core region has been observed at increasing the rotating rate. The study concludes some distinct characteristics and also some common features with the classical spanwise rotation case.

Introduction

The development of compact and efficient turbomachinery has been accelerated during the last two decades. Consequently, the researches have been directed to the flows subjected to system rotation in order to explore the flow characteristics and detailed information on the effect of Coriolis and centrifugal forces on the flow fields. The simple and straightforward model to study the rotating flows was the two-dimensional channel flow, in which the rotating axis coincides with the spanwise direction. This model resembles the flow inside radial rotors. Although the model is far from reality due to neglecting curvature, tilting of blades, and contraction of flow passages, it has been studied experimentally and numerically for the last four decades. The pioneer experimental works were that Hill, Moon and Moore during the 60s in the MIT gas turbine laboratory. Johnston and his co-workers conducted the experimental research in (1966) by studying the long rotating rectangular channel and his experiment in (1972) had revealed more detailed measurements and showed the change in turbulence levels along the two sidewalls of the channel. Using LES and DNS, the numerical investigations became feasible with the aid of super computers and large data managements. Kim (1983) used LES to study the Poiseuille flow in a rotating channel and in (1993); Kristoffersen et al applied DNS by the means of finite difference techniques to solve the spanwise rotating channel at different rotating numbers. The numerical studies verified the observations of the earlier experiments and have given a reliable data bases in which the instantaneous flow fields can be visualized and different terms in the budget of the transport equations can be determined up to the wall.

In reality, the flow in radial turbines passages has no unique direction and even the walls of the blades are not totally perpendicular to the plan of rotation, which means neither the rotating axis would be appended to the spanwise direction nor the flow direction would be perpendicular to the axis of rotation. In other words, having the rotating axis with some phase with the Cartesian coordinates may be more realistic and practical than the fundamental spanwise case. In (1997) in CTR, Oberlack et al. started a comprehensive study using different approaches in order to investigate the effect of streamwise rotation, in which the axis of rotation coincides with the flow mean direction, see also Oberlack et al (1999). The objective of this study is to conduct the work of Oberlack to include wider range of rotation number and to compare between the two cases namely; the classical spanwise, and streamwise cases. Generalizing the rotating vector orientation nowadays becomes a necessarily demand in order to improve the existing turbulence models used in the design of compact gas turbines.

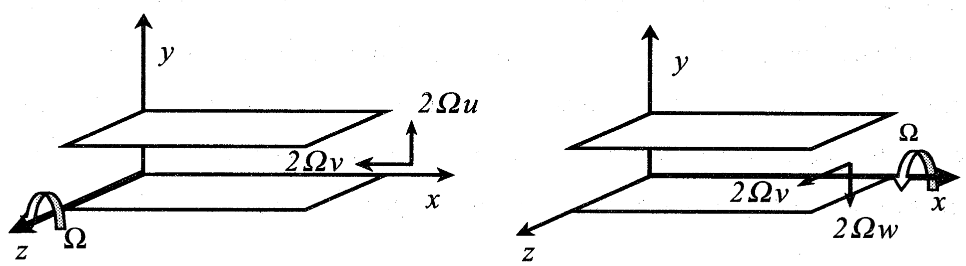


Figure 1 (a) Spanwise rotation (classical case), and (b) streamwise rotating case

Numerical analysis

Using Pseudospectral method, the full Navier-stokes equations are solved for a rotation number ranged from 0 to 10 in the spanwise case, while the range is extended to 15 in case of streamwise rotation. 64 grid points have been used in all the directions, where the Fourier series are used in the spanwise and streamwise directions and Chebyshev in the wall-normal direction. More details about the numerical technique can be reviewed elsewhere for example Kim et al (1987), Kuroda (1990) and Kuroda et al in (1990) and (1995). The geometry is shown in figure 1 in which the different rotating axes and the resulted Coriolis forces can be distinguished.

The continuity and Navier-Stokes equations for the streamwise case can be written in dimensionless form as follows in which the Reynolds and rotation numbers are defined based on the channel half width and friction velocity u_τ;

$$
\begin{aligned}
&\frac{\partial u_j}{\partial x_j}=0,\\
&\frac{\partial u}{\partial t}=-\frac{\partial P}{\partial x}-u_j\frac{\partial u}{\partial x_j}+\frac{1}{\mathrm{Re}_\tau}\frac{\partial^2 u}{\partial x^2},\\
&\frac{\partial v}{\partial t}=-\frac{\partial P}{\partial y}-u_j\frac{\partial v}{\partial x_j}+\frac{1}{\mathrm{Re}_\tau}\frac{\partial^2 v}{\partial x^2}-\underline{Ro_\tau w},\\
&\frac{\partial w}{\partial t}=-\frac{\partial P}{\partial z}-u_j\frac{\partial w}{\partial x_j}+\frac{1}{\mathrm{Re}_\tau}\frac{\partial^2 w}{\partial x^2}+\underline{Ro_\tau v},
\end{aligned}
\tag{1}
$$

The underlined terms are the Coriolis components in the wall-normal and spanwise directions. Since the wall-normal term is function of the spanwise velocity, it is expected that the order of magnitude of this term to be less than the corresponding one in spanwise rotating case in which the Coriolis term was function of the streamwise velocity. It is convenient to spot some light on the mean momentum equations and the production terms resulted from streamwise rotation and compare them with the classical case in order to understand better the difference between the two cases. The symmetric characteristic of the governing equations yields the conclusion of symmetric profiles for mean streamwise velocity U_m, while asymmetric for the spanwise component W_m.

Table 1 Mean momentum equations

Spanwise Rotation	Streamwise Rotation
$0=-\frac{\partial P}{\partial x}-\frac{d(\overline{uv})}{dy}+\frac{1}{\mathrm{Re}}\frac{d^2U}{dy^2}$ $0=-\frac{\partial P}{\partial y}-\frac{d(\overline{v^2})}{dy}+2\Omega_3 U$	$0=-\frac{\partial P}{\partial x}-\frac{d(\overline{uv})}{dy}+\frac{1}{\mathrm{Re}}\frac{d^2U}{dy^2}$ $0=-\frac{\partial P}{\partial y}-\frac{d(\overline{v^2})}{dy}-2\Omega_1 W$ $0=-\frac{d(\overline{vw})}{dy}+\frac{1}{\mathrm{Re}}\frac{d^2W}{dy^2}$

Table 2 Production terms due to rotation G_{ij}

	Spanwise rotation	Streamwise rotation
G_{11}	$4\Omega_3\overline{uv}$	0
G_{22}	$-4\Omega_3\overline{uv}$	$4\Omega_1\overline{vw}$
G_{33}	0	$-4\Omega_1\overline{vw}$
G_{12}	$-2\Omega_3(\overline{u^2}-\overline{v^2})$	$2\Omega_1\overline{uw}$
G_{23}	0	$-2\Omega_1(\overline{w^2}-\overline{v^2})$
G_{13}	0	$-2\Omega_1\overline{uv}$

The same property can be deduced for the Reynolds stresses components in their transport equations, which are not shown here, in which the $\overline{uv}$, and $\overline{uw}$ are asymmetric while $\overline{vw}$ is symmetric. It evident now that the Coriolis force in the wall-normal direction in the mean would be of less effect than that of classical case. The other Coriolis term in the spanwise direction would be responsible of some sort of lateral motion, by which the off-diagonal Reynolds stresses would be non-zero and in sequence, there would be a non-zero spanwise velocity as shown in the *z*-equation. From table 2, we can see the additional production terms resulted from system rotation, which resulted in the non-zero Reynolds stresses. It can be deduced that the production terms due to mean shear will also be affected indirectly by system rotation due to the non-zero gradient of mean spanwise velocity.

Results and Discussion

a. Spanwise rotation

A quick review of the classical spanwise rotation at higher rotation numbers than that of Nishimura et al (1996) will be given at first, before presenting the streamwise case. A comparison at the same rotating number for the two cases will be also shown. Stabilization near the suction side at $y/\delta = -1$ is clearly recaptured. Linear profiles at the centre of the channel have been observed.

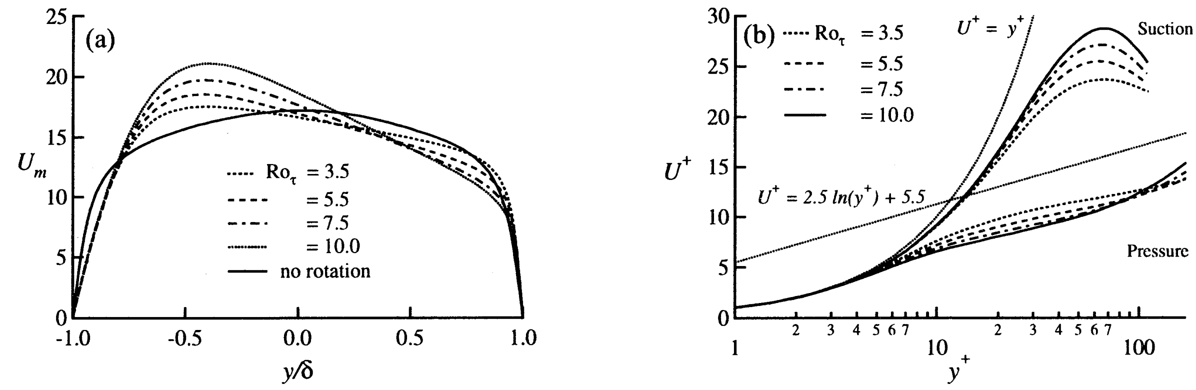


Figure 2 (a) Mean velocities for higher rotation numbers, and (b) Mean velocities profiles at suction and pressure walls in wall units.

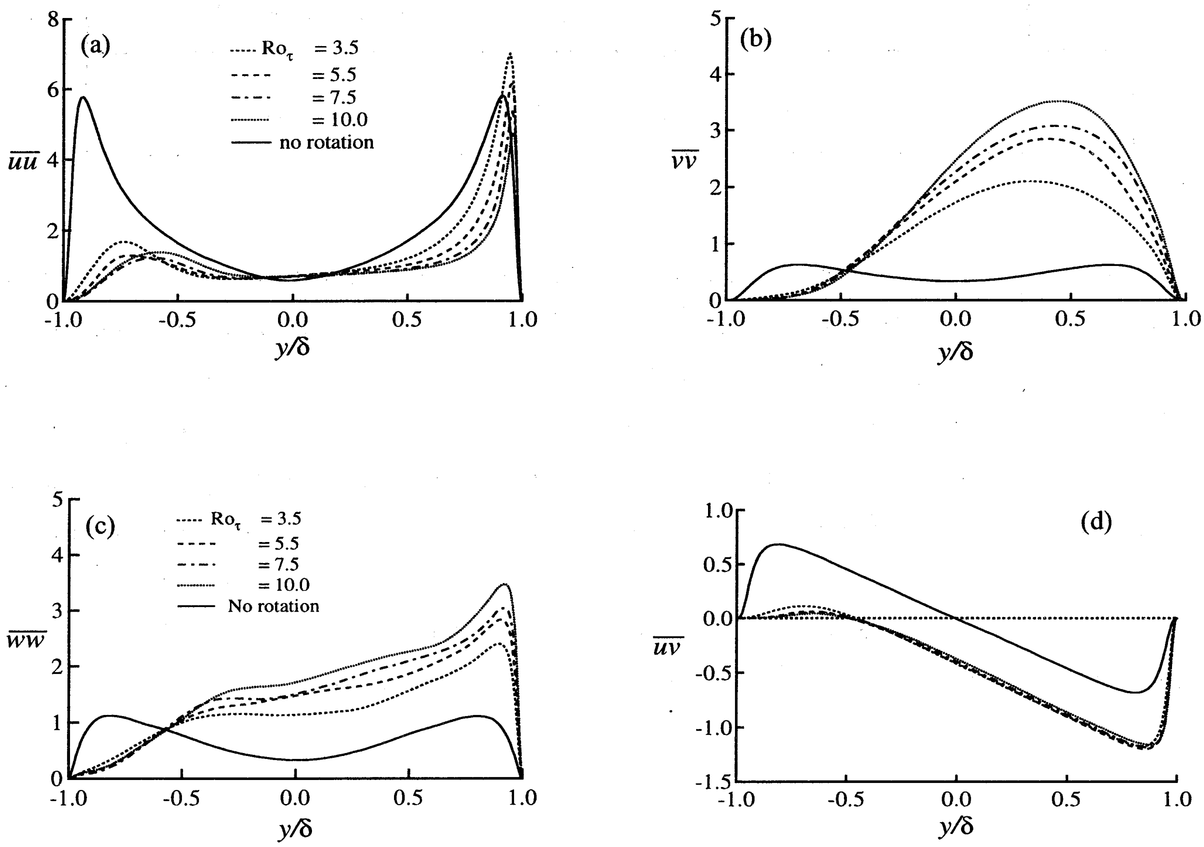
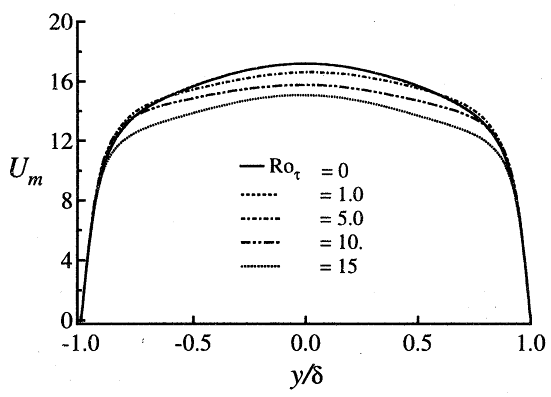


Figure 3 Reynolds stresses components for higher rotation number; (a) $\overline{uu}$, (b) $\overline{vv}$, (c) $\overline{ww}$, and (d) $\overline{uv}$

The Reynolds stresses also are very well captured compared with those of Kristoffersen et al (1993) in which the stabilization zone shows lower levels of turbulence intensities. Suppression of turbulence near the suction side yields to relaminarization and the Reynolds stresses approaching zero as the rotation number increases.

b. Streamwise rotation

The mean velocities in stream and spanwise directions are shown in figure 4, in which the symmetric and asymmetric profiles can be recognized for both respectively. The discrepancies in the profiles around the centreline are due to short average time periods. This type of rotation needs longer time for statistical averaging as well as wider computational domain as will be shown in the two point correlations later. More streamwise flatten profiles of velocities accompanied with lower mass flow rate have been observed with increasing the rotation number. Four layers of opposite motion can be observed in the streamwise velocity and the gradient of the velocities would grow up with increasing the rotation, which in turns enhance the Reynolds stresses as shown previously in Table 2.



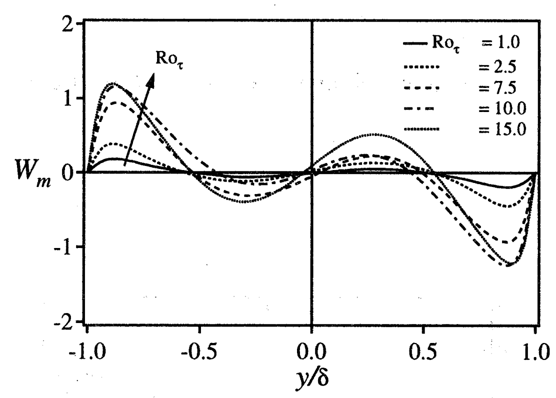


Figure 4 Left; mean streamwise velocity, right mean spanwise W_m velocity component for wide range of rotation numbers

The Reynolds stresses in streamwise case show little changes to the stationary case at low rotation numbers, and the effect will be obvious for higher rate ($Ro_\tau \geq 5$). No laminarization zones will be arisen and the normal stresses shown in figure 5 show turbulence enhancement within the whole channel width and significant increment in spanwise component is observed near the wall. The non-zero off-diagonal Reynolds stresses are shown in figure 6a and the effect of increasing the rotation number can be observed in figure 6b in which effect near the wall is clearly observed.

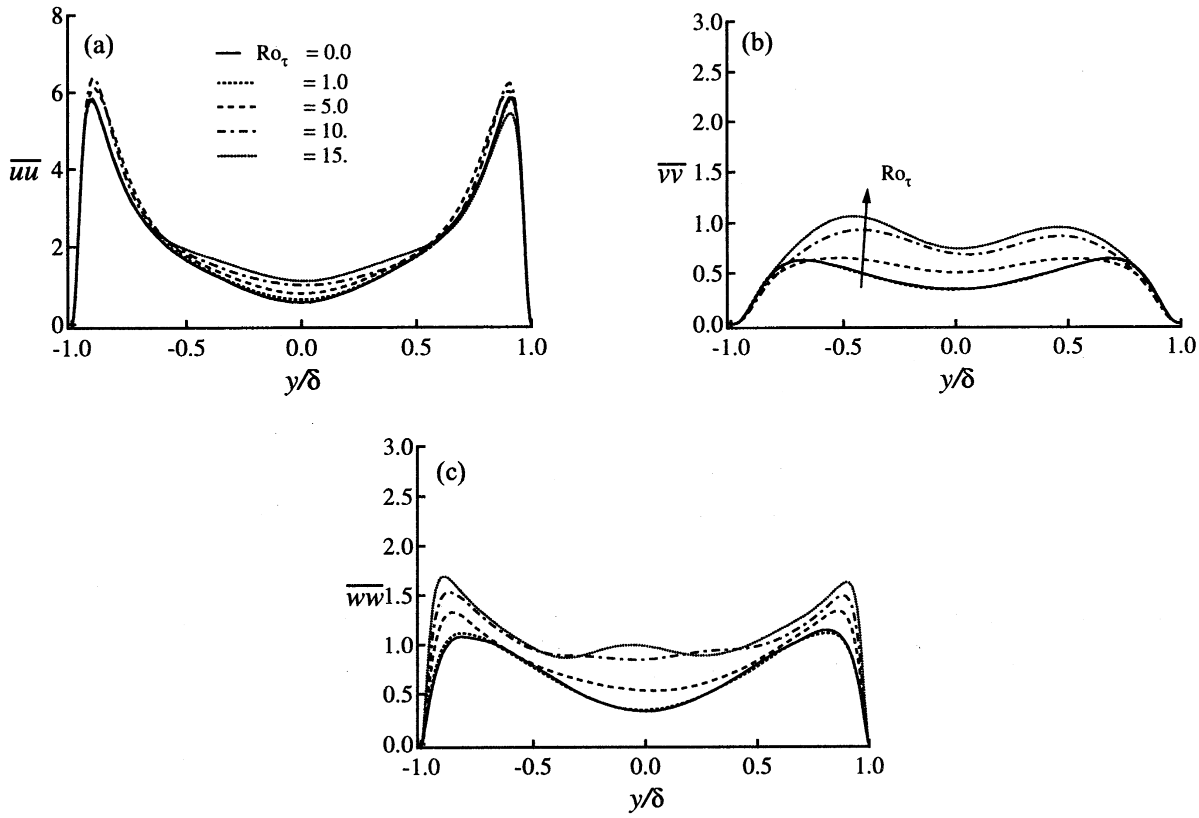


Figure 5 Normal stresses in streamwise rotation; (a) $\overline{uu}$, (b) $\overline{vv}$, and (c) $\overline{ww}$ component

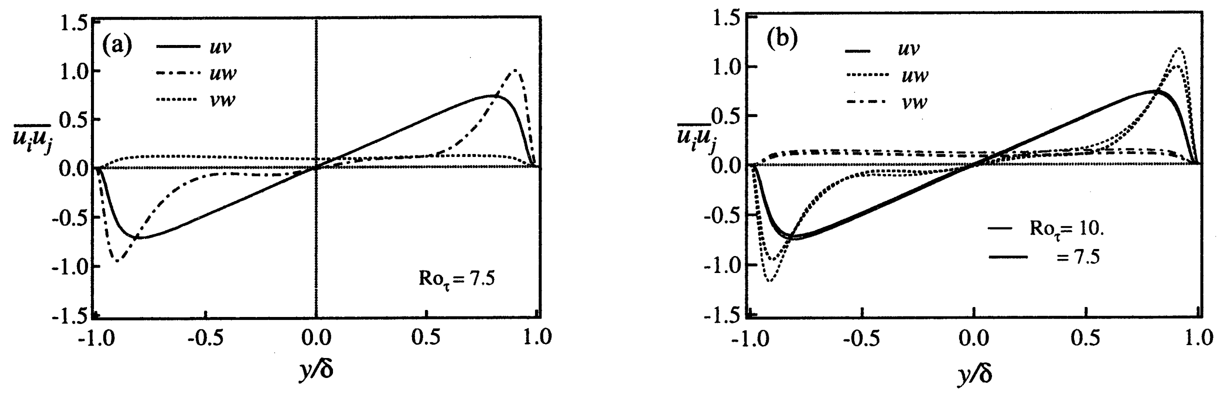


Figure 6 Reynolds shear stresses; (a) at $Ro_\tau = 7.5$, and (b) at rotating numbers; 7.5 and 10.

The $\overline{uw}$ component in figure 6b shows higher values near the wall because the large difference between the $\overline{ww}$ and $\overline{vv}$ locally in near wall region that appears in table2. This difference is not that much in the core that resulted in low values of stress within the range of $y/\delta = \pm 0.5$.

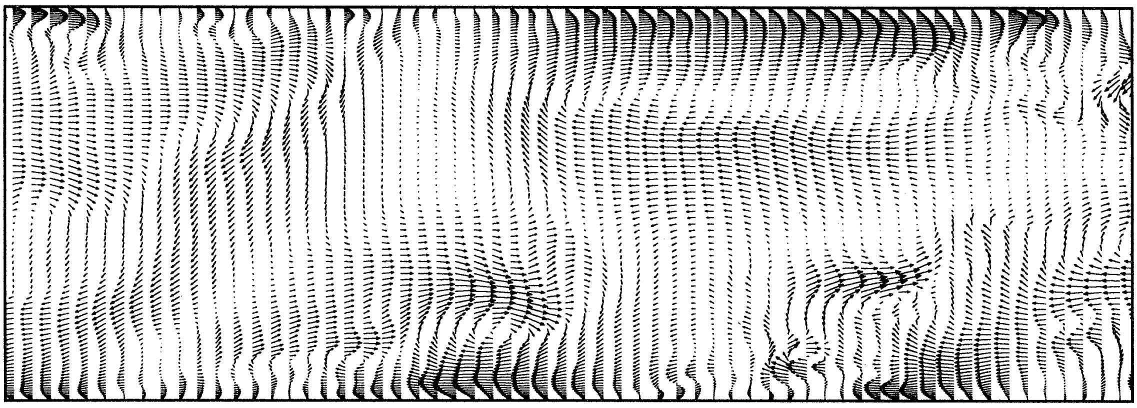


Figure 7 Vector plot of the cross-stream velocity components at $Ro_\tau = 10$.

The lateral motion shown in figure 8 indicates the effect of streamwise rotation upon the flow filed especially near the walls. Finer grids may be needed to capture more detailed information about vortical structure. The visualized streaky structure, not shown here, shows inclined structure rather than that of spanwise rotation. But no clear picture of low-high speed streaks could be observed. Two-point correlations in figure 9 show non-zero values at separation of half the stream and spanwise lengths and longer dimensions are seen necessary to well resolve the rotating flows with different rotating axes .

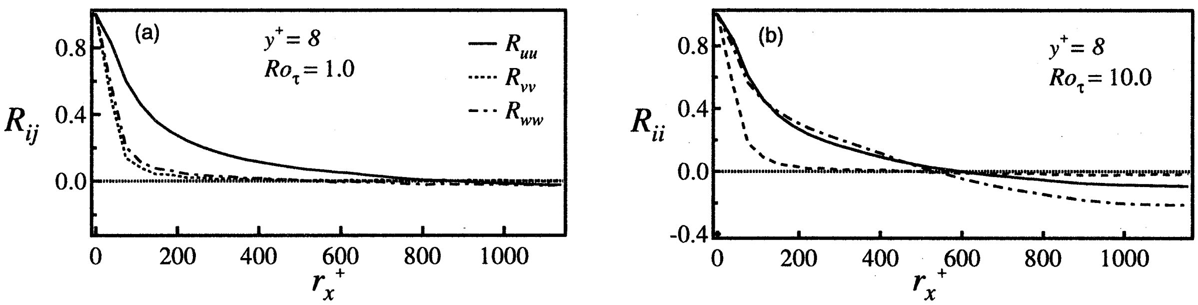


Figure 8 Two point correlations in spanwise direction at 8 wall units far from wall;

(a) $Ro_\tau = 1.0$, and (b) $Ro_\tau = 10.0$

Conclusions

DNS have been performed in order to explore the behavior of channel flow subjected to streamwise rotation as an intermediate step towards the generalization of the problem. Streamwise and normal to wall rotation cases induce additional production of turbulence due to rotation. Symmetric profiles of U_m and asymmetric profiles of W_m and have been verified. All the Reynolds stress components are non-zero for both streamwise and normal to wall cases and spanwise mean velocity is induced as a result of the non-zero component. At low streamwise rotation, Ro less than 5, weak effects on mean velocity and normal Reynolds stresses due to the small gradient of spanwise mean velocity and weak correlations of shear stresses. Four layers of opposite spanwise motion have been recognized with different thickness, which augment the turbulence in the core region. Larger domain is needed while increasing the rotation rate. Future work may include finer grid resolution in order to analyze the coherent structure and turbulence statistics.

Since the features of streamwise rotation differ from that of classical spanwise case, it would a challenge for turbulence modeler to develop a general model for different rotation axes. Studying the effect of general rotating axes becomes important in terms of simulating the real turbomachines and validates the turbulence models.

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